A Method for Computing Float-Platform Motions in Waves

John A. Mercier*
Stevens Institute of Technology, Hoboken. N. J.

A new procedure is presented for the prediction of the longitudinal motions, in regular head seas, of ocean platforms supported by slender vertical floats. Experimental information describing the hydrodynamic forces acting on the floats is used to solve the equations of motion for harmonic oscillation in heave, surge, and pitch. These hydrodynamic forces are obtained from tests conducted on a single isolated float. They are expressed as two separable parts, one due to the disturbance imposed by the waves acting as though the vehicle were completely restrained and one due to the motion of the vehicle as though there were no wave disturbance. When the measured motions of a free four-float vehicle are compared with the motions as computed under the new procedure, reasonably good agreement is found. The approach should be useful in design development and in structural load evaluation not only for multi-float vehicles but for other types of marine structures as well.

Nomenclature

$\bar{A}_{ij}$ = coefficient (amplitude and phase) expressing force in ith direction due to jth mode of motion per unit motion
a = vertical distance from platform c.g. to origin of coordinates in individual float
$\bar{B}_l$ = coefficient (amplitude and phase) expressing amplitude of lth component of force due to wave passage
b = horizontal distance from platform c.g. to origin of coordinates in individual float
d = diameter of float
F_{ij} = force (or moment) in ith direction due to jth mode of motion
f = frequency of motion
g = acceleration of gravity
$2h_{zz}$ = damping coefficient
I_y = mass moment of inertia of body about y (transverse) axis
k_{zz} = added mass coefficient
l = distance from wave measurement location to origin of coordinates in float
M = mass of body
s = distance from wave measurement location to platform c.g.
t = time
x,z,θ = surge, heave, and pitch displacement at platform, respectively
ξ,ζ,θ = body-fixed coordinates used to measuring forces acting on floats
α_i = phase between wave passage and ith mode of platform motion
β_{ij} = phase between ith component of force and jth mode of motion, due to jth mode of motion in otherwise undisturbed water
γ_l = phase between wave passage and lth component of wave-induced force
η = wave elevation

Subscript

k = values for the kth float of a multifloat platform

Introduction

THE use of slender vertical floats to support a platform above the surface of the sea can afford a comfortable and stable working platform with remarkably small motions even in very heavy seas. This type of flotation system has many applications; it can be used for sea-based aircraft, helicopters, sea sleds, floating platforms or buoys, ground effect machines, etc. A thorough discussion of the concept and its development, as applied to aircraft for open-ocean operation, has been presented by Handler.[1] Other naval uses have been considered by Jayne[2] and a recent marine drilling rig directory[3] illustrates a number of floating platforms whose buoyancy is provided by vertical floats.

Experiments to evaluate the motion-response characteristics of a vehicle floating on the ocean's wavy surface are ordinarily conducted with scale models whose hydrodynamic and mass dynamic characteristics are similar to those of the full-scale vehicle. Extensive systematic experimental work of this sort has been carried out at Davidson Laboratory for vertical float supported craft, as reported by Mercier.[4]

An alternative procedure has been developed for studying platform motions, where the hydrodynamic qualities can be separated from the inertial qualities. This paper describes an exploratory investigation of this procedure which also makes the investigation of such features as float arrangements and spacing more convenient and less time consuming and expensive. It is in some respects related to an analysis by Barr and Martin.[5] That work, however, attempts to construct hydrodynamic-force inputs on the vehicles without experimental verification for the float configurations themselves. It synthesizes theoretical and experimental results from studies unrelated to float vehicles, for application to the calculation of the motions of actual vehicles. It is believed that the present approach is more securely founded on experiment and involves simpler numerical calculations.

The method as described below is used to evaluate the motions-response characteristics of a particular cruciform float vehicle. The predicted motions are compared with motions which had been measured previously and reasonably good agreement is found.

Method

The motion of a float platform or other vehicle in a seaway can be determined by the solution of the system of differential equations of motion. Primarily, this paper is concerned with the force inputs to these equations, for the case of float platforms. For a vehicle floating in waves, the forces are hydrodynamic in nature and may be thought of as being composed of two separable parts, one due to the disturbance imposed by the waves acting as though the vehicle were completely restrained, and one due to the motion of the vehicle as though there were no wave disturbance.

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* Research Engineer, Davidson Laboratory.

For present purposes, the hydrodynamic forces are derived from experiments conducted with a single, isolated float. The fact that many floats may be connected to support a platform is taken into account through the assumption of rigid-body kinematics relating the individual float motions. Hydrodynamic interactions between the supporting floats are assumed to be negligibly small, which is not unreasonable for slender, relatively wide-spaced floats.

When the forces due to waves are assumed to be independent of, or separable from, the forces due to float motion (which is caused by the waves), a simplification is effected which is probably justified for small motions. In this case, the equations of motion may be linearized. Thus, the exciting forces due to waves are assumed to be proportional to the wave height and the forces due to the motion of floats in smooth water are assumed proportional to the motions. The forces are described by their amplitude and phase relative to the waves or motions and are assumed to be frequency dependent—but simple harmonic in time—for regular waves and simple harmonic motions. With these assumptions and with experimentally determined values for the necessary hydrodynamic forces, the differential equations of motion reduce to algebraic equations which can readily be solved.

Justification for the assumptions is best established by comparing the results of computations carried out by using the proposed method with test results for the motion of a free model in waves. This comparison is discussed later for one platform configuration.

Although the method is given for regular waves, it is, of course, possible to derive the response to a more realistic long-crested irregular sea by using either a deterministic or a statistical description of the vehicle's behavior.[7] Procedures for evaluating the responses to irregular waves by the use of the transfer functions derived according to the methods of this paper will not be discussed in detail; they are available elsewhere.

Analysis

Float Forces

X_m, Z_m, and Θ_m are hydrodynamic forces associated with the motion of the float in smooth water determined from a series of tests with the float oscillated in a variety of combinations of pitch, surge, and heave motion and described by the equations below.

$$\begin{aligned} X_m &= \bar{A}'_{xx}x + \bar{A}'_{xz}z + \bar{A}'_{x\theta}\theta \\ Z_m &= \bar{A}'_{zx}x + \bar{A}'_{zz}z + \bar{A}'_{z\theta}\theta \\ \Theta_m &= \bar{A}'_{\theta x}x + \bar{A}'_{\theta z}z + \bar{A}'_{\theta\theta}\theta \end{aligned} \tag{1}$$

The coefficients $\bar{A}'_{ij}$ (amplitude and phase) of these equations will be determined for a number of values of frequency. It should be borne in mind that x, z, and θ, the surge, heave, and pitch displacements, respectively, are not generally in phase with one another. Furthermore, for floats with longitudinal and transverse symmetry, a vertical motion should give rise to no surge or pitch force (for the origin of coordinate axes on the axis of symmetry); hence $\bar{A}'_{xz} = \bar{A}'_{\theta z} = 0$.

The forces due to the motions are assumed to be linear and superposable as Eq. (1) indicates (the assumption can be checked by experiment). These forces can be expressed in terms of nondimensional coefficients which are assumed to depend on the frequency parameter $\omega(d/g)^{1/2}$, where ω is the frequency of harmonic motion, d is the float diameter (or other significant dimension), and g the acceleration of gravity. Then, for example, the vertical force due to a vertical motion F_{zz} can be expressed as

$$F_{zz} = \Delta(\omega^2/g)z\bar{A}_{zz}\omega(d/g)^{1/2} \tag{2}$$

where Δ is the float displacement and $\bar{A}_{zz}$ is a dimensionless complex function of $\omega(d/g)^{1/2}$. The pitch moment due to a surge motion $F_{\theta x}$ can be given similarly by

$$F_{\theta x} = \Delta d(\omega^2/g)x\bar{A}_{\theta x}\omega(d/g)^{1/2} \tag{3}$$

where $\bar{A}_{\theta x}$ is again a dimensionless complex function of the frequency parameter $\omega(d/g)^{1/2}$.

The forces (amplitude and phase) due to the wave disturbance on a single float are determined (see below) from an experiment on a restrained model in regular waves. These forces are proportional to wave height and are dependent on wave frequency.

For a water wave of length λ progressing in the $-x$ direction and measured at the longitudinal location x, the wave elevation is given by

$$\eta = \eta_0 \exp i(\omega t + 2\pi x/\lambda) \tag{4}$$

The wave is measured by a sensor located at a distance l from the float and the vertical component of the force which the wave induces on a float can be expressed as

$$Z_d = \Delta(\eta_0/d)\bar{B}_z\omega(d/g)^{1/2}e^{i\omega t}\exp\left[-i(\omega^2 d/g)(l/d)\right] \tag{5}$$

where $\bar{B}_z$ is a dimensionless complex function of the frequency parameter. Thus, wave-induced forces acting on a float can be determined in terms of wave measurements obtained at some other location.

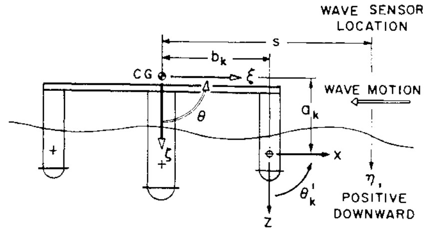

Fig. 1 Multifloat platform with associated coordinate systems.

Equations of Motion

The linearized equations of motion for a multifloat platform, such as is illustrated in Fig. 1, are to be given in terms of a system of coordinates fixed at the center of gravity of the vessel with body-fixed axes coincident with the principal axes of inertia. The notation is similar to that given in standard nomenclature.[8] The pitch θ, heave ζ, and surge ξ oscillations of a float system with transverse symmetry in head seas are treated. The input information is the wave elevation, assumed at a point a fixed distance s forward of the origin of the coordinates. The subscript k denotes values for the kth float and the bars over the symbols indicate complex quantities. For harmonic motions of the form

$$\bar{\xi} = \xi\exp[i(\alpha_\xi + \omega t)],\ \bar{\zeta} = \zeta\exp[i(\alpha_\zeta + \omega t)]$$
$$\bar{\theta} = \theta\exp[i(\alpha_\theta + \omega t)] \tag{6}$$

the equations of motion for heave, surge and pitch can, after linearization, be written as follows:

heave equation:

$$M(-\omega^2\zeta e^{i\alpha_\zeta}) = \sum_k \Delta_k\bar{B}_{zk}\frac{\eta_0}{d_k}\exp\{-i(\omega^2 d_k/g)[(s - b_k/d_k)]\} + \sum_k\frac{\omega^2}{g}\Delta_k\bar{A}_{zxk}\bar{x}_k + \sum_k\frac{\omega^2}{g}\Delta_k\bar{A}_{zzk}\bar{z}_k + \sum_k\frac{\omega^2}{g}\Delta_k\bar{A}_{z\theta k}\bar{\theta} \tag{7}$$

surge equation:

$$M(-\omega^2\xi e^{i\alpha_\xi}) = \sum_k \Delta_k\bar{B}_{xk}\frac{\eta_0}{d_k}\exp\{-i(\omega^2 d_k/g)[(s - b_k)/d_k]\} + \sum_k\frac{\omega^2}{g}\Delta_k\bar{A}_{xxk}\bar{x}_k + \sum_k\frac{\omega^2}{g}\Delta_k\bar{A}_{xzk}\bar{z}_k + \sum_k\frac{\omega^2}{g}\times \Delta_k d_k\bar{A}_{x\theta k}\bar{\theta} - \Sigma\Delta_k\theta \tag{8}$$

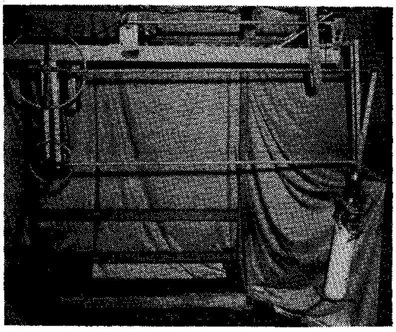

Fig. 2 Pitch, heave, and surge oscillator with force - balance system and model.

pitch equation:

$$I_y(-\omega^2\theta e^{i\alpha_\theta}) = \sum_k (\Delta_k b_k \bar{B}_{zk} + \Delta_k a_k \bar{B}_{xk} + \Delta_k d_k \bar{B}_{\theta k}) \cdot \frac{\eta_0}{d_k} \exp\{-i(\omega^2 d_k/g)[(s - b_k)1d_k]\} + \sum_k \frac{\omega^2}{g} \Delta_k d_k(\bar{A}_{\theta x k} x_k + \bar{A}_{\theta z k} z_k + d_k \bar{A}_{\theta\theta k}\bar{\theta}) - \sum_k \frac{\omega^2}{g} \Delta_k b_k(\bar{A}_{zxk}\bar{x}_k + \bar{A}_{zzk}\bar{z}_k + d_k \bar{A}_{z\theta k}\bar{\theta}) + \sum_k \frac{\omega^2}{g} \Delta_k a_k(\bar{A}_{xxk}\bar{x}_k + \bar{A}_{xzk}\bar{z}_k + d_k \bar{A}_{x\theta k}\bar{\theta}) \quad (9)$$

where

$$\bar{x}_k = \bar{\xi} + a_k\bar{\theta}, \ \bar{z}_k = \bar{\zeta} - b_k\bar{\theta} \quad (10)$$

The force coefficients are written explicitly in the form

$$\bar{A}_{ij} = A_{ij}\exp(i\beta_{ij}), \quad \bar{B}_l = B_l\exp(i\gamma_l) \quad (11)$$

Equations (7–9) can be solved algebraically to determine the amplitudes and phase relations of the resulting motions. The phase angle measures the fraction of a cycle between the occurrence of maximum motion and the movement of a wave trough past the wave sensor location with a positive value signifying that the motion maxima occur after the wave maxima.

The surge equation (8) differs from the corresponding equation given in an earlier paper by the author.[6] The side force due to the body weight

$$\sum_k \Delta_k\theta$$

has been incorporated because it must be included when a body-fixed coordinate system is used.

Experiments

Apparatus

The oscillatory forces acting on a single float were determined for two conditions: 1) with the model held fixed in regular waves; 2) with the float oscillated in otherwise undisturbed water so as to produce combinations of pitch, heave, and surge motions.

A simple machine was devised to produce the required combinations of motions. The frequency of the motions was controlled by adjusting the speed of a geared *d-c* motor. The model was connected to the machine's driven element through a system of balances which measured heave force, surge force, and pitch moment about a prescribed axis. This same device supported the model firmly for the tests in waves.

Figure 2 is a photograph of the oscillator. The range of possible motions included heave, $\pm 1\frac{3}{8}$ in. max, in increments of $\frac{1}{8}$ in.; surge, ± 2 in. max, in increments of $\frac{1}{4}$ in.; and pitch, $\pm 10°$ max, in increments of $1°$.

Test results are in the form of oscillograms of the force signals, suitably amplified and filtered to reduce high-frequency noise content. The motion of the model was recorded by the output of a rotary potentiometer used as a position transducer. This motion record was essential for determination of the phase relation between force and motion. For tests of the fixed model in waves, a resistance type of wave wire was mounted in line with the model, approximately midway between the model and the sidewall of the tank. Wave records give both the wave amplitude and the phase information which relates the force and the passage of the wave trough.

The tests were conducted in Davidson Laboratory's Tank 3 which is 300 ft long, 12 ft wide, and 6 ft deep. The model and apparatus were attached to the tank rail structure at a location about 75 ft from the wave generator. They have no forward speed.

Model

The model studied in these tests is illustrated in Fig. 3. The same float was used in earlier tests of a four-float dynamic model of a float-supported aircraft. Those tests were conducted with the dynamic model floating freely in regular seas to determine the motion response of the vehicle in pitch, heave, and surge for a range of frequencies. Below, the measured motions of the free model and the predicted motions (obtained by using present force measurements and procedures) will be compared.

The system of body-fixed axes for the float is shown in Fig. 3. Pitching moments about the y(transverse) axis were determined and pitch motions about this axis were produced by the oscillator. The forces were measured relative to the body-fixed coordinate system.

The model is made of polyurethane foam with a thin-walled aluminum tube inserted down the center of the cylindrical portion to provide extra rigidity. The damping plate is made of $\frac{1}{8}$-in. thick plexiglass with sharply bevelled edges.

Test Procedure and Program

The apparatus was secured to the rail of the towing tank and the force balances and wave wire were calibrated. Tests were conducted in regular waves of varying frequency with peak-to-peak amplitudes of approximately 2 in. Enough signal cycles were recorded to assure steady-state conditions and to allow averaging for more accurate analysis. Approximately 5 min were allowed between tests in waves of different frequency to allow the waves to decay and the water surface to become smooth.

For tests in smooth water with forced oscillation of the model, the position transducer was calibrated to correlate motion and oscillogram signal. Motor speed was set and a series of tests was run with different modes of oscillation and with different amplitudes. An adequate number of signal cycles were recorded as before. Tests with combinations of

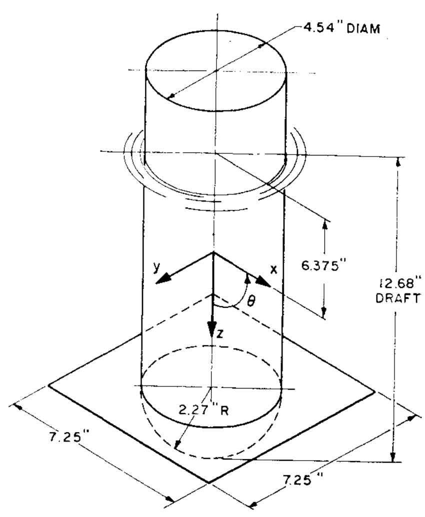

Fig. 3 Float model and coordinate system.

pitch and heave and of surge and heave were conducted to learn whether or not important interactions would occur. Frequency range was limited on the low-frequency end by the inability of the motor speed control to provide harmonic motion for low speeds and was limited on the high-frequency end by forced vibrations of the oscillator in the sidewise (across the tank) direction.

Correction of Data

Since we want to describe those hydrodynamic forces (acting on the float) which are because of the wave impingement and forced oscillation on smooth water, certain of the measurements made in the towing tank must be corrected to account for inertial and other forces not associated with fluid mechanics. These corrections apply only to forces due to forced oscillation. For the heave and surge motions, the forces are basically purely inertial, but for the pitch motion, the forces were measured in body-fixed axes and, therefore, a weight component of the correction for side force and pitch moment must be dealt with. The corrections were based upon tests of the model and apparatus conducted in air. Graphical vector subtraction was used to account properly for the phases of the forces.

Results

An example of the reduced data, without correction, is shown in Fig. 4 for pure heave motion of three different amplitudes. The force is divided by the amplitude of motion and the square of the frequency so that the range of numerical values will be suitably small. This force coefficient and the phase are plotted against frequency of oscillation. The figure shows the degree of scatter and indicates that the assumption of linearity of force and motion is reasonably safe. Other data are of similar quality and the results of tests with combinations of motions support the assumption of the linear dependency of the forces on the motion.

The (corrected) hydrodynamic forces and their phase angles, both of which are associated with the various motions of the float, are presented in Figs. 5a and 5b in nondimensional form. The coefficients, defined by the nondimensionalizing procedure indicated below the figure, are plotted vs frequency parameter $\omega(d/g)^{1/2}$. Because of symmetry, the coefficients $A_{xz} = A_{\theta z} = 0$ since vertical motion of the float does not produce a side force or pitch moment. And for this float, it has been found that surge motion and pitch motion produce only negligibly small vertical forces; hence, $A_{zx} = A_{z\theta} = 0$ in this case.

The forces due to pitching motion $(A_{x\theta}, A_{\theta\theta})$ cannot be evaluated with significant accuracy from the measurements taken because the hydrodynamic forces are small compared

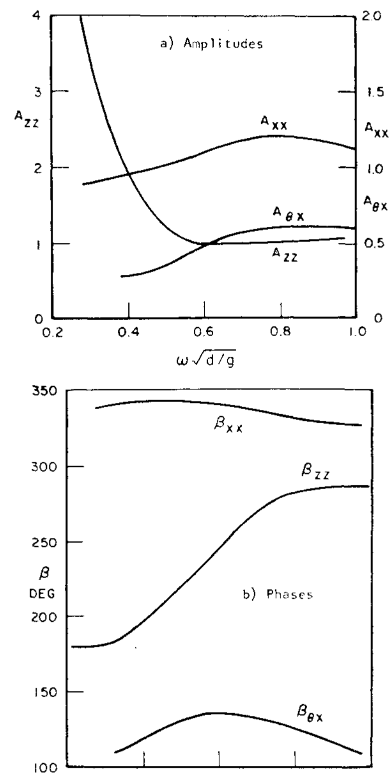

Fig. 5 Hydrodynamic forces due to float motions:

$$A_{zz}e^{i\beta_{zz}} = (\text{heave force}/z\ \text{motion})\cdot g/\Delta\omega^2$$
$$A_{xx}e^{i\beta_{xx}} = (\text{surge force}/x\ \text{motion})\cdot g/\Delta\omega^2$$
$$A_{\theta x}e^{i\beta\theta_x} = (\text{pitch moment}/\theta\ \text{motion})\cdot g/\Delta d\omega^2$$

to the inertial and weight corrections. It is expected that the hydrodynamic side force due to pitch motion will be due primarily to the buoyant force, but will also contain some damping force and added-mass force. In an approximation for use in computing platform motions, the side force will be estimated as that due to buoyancy only, viz., $A_{x\theta} = (\Delta\theta)g/(\theta\Delta\omega^2 d) = g/d\omega^2$ and $\beta_{x\theta} = 0$. The pitch moment due to pitch motion is principally due to inertial and damping forces, at least for this particular float and for the coordinate system adapted. It is expected that this moment will prove unimportant for evaluating float-platform motion and $\bar{A}_{\theta\theta}$ is assumed equal to zero. These coefficients were incorrectly reported earlier[6] due to an error in data reduction.

The force coefficients in Fig. 5 are given in the form of amplitude and phase because it is convenient to use them in this fashion to calculate platform motions from Eqs. (7–9). The added mass and the damping component of the vertical force due to vertical motion are shown in Fig. 6 where they are compared with theoretical results obtained by Havelock[9] for a semisubmerged heaving sphere. The component of the heaving force in phase with the motion is associated with buoyancy and added mass and the out-of-phase component is the damping force. The frequency parameter of Fig. 6 is $\omega^2 a/g$ (as used by Havelock) where a is the radius of the sphere or an equivalent dimension for the float $(3 \times \text{displ. vol.}/2\pi)^{1/3}$; therefore, frequencies are equal for equal displacements of the float and the sphere. The added-mass coefficient ($\bar{k}_{zz}$ = added mass $\times\ g/\Delta$) is seen to be frequency dependent for both sphere (theoretical) and float with damping plate (experimental) with the maxima occurring at significantly different frequencies. For the damping coefficient ($2h_{zz}$ = out-of-phase force per unit motion $\times\ g/\Delta\omega^2$), the float shows a substantially greater value than does the sphere, but both are frequency dependent. The greater out-of-phase force for the float is due to the sharp-edged plate which contributes a great deal of viscous damping over and above that which is due to wave generation.

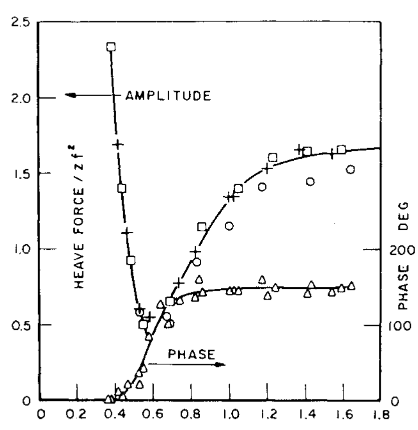

Fig. 4 Example of uncorrected data for heave force associated with heave motion.

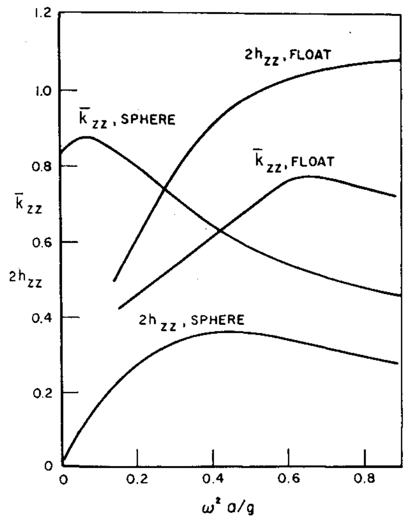

Fig. 6 Added mass and damping coefficients for the float model and for a hemisphere oscillating vertically in the free surface.

The forces due to wave disturbance, and their phase angles relative to the passing of a wave trough, are shown in Fig. 7a and 7b. The nondimensional coefficients are not defined in exactly the same way as those used for the motion results since the forces do not become large at high frequencies. The definitions of the coefficients are given below the figure. They also are plotted vs the frequency parameter $\omega(d/g)^{1/2}$.

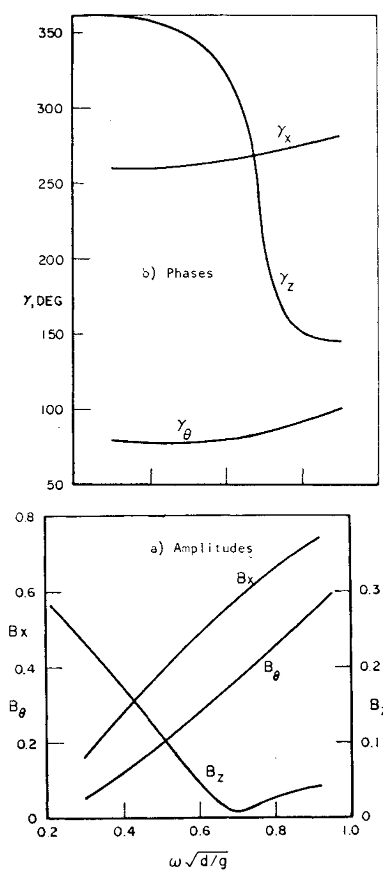

Fig. 7 Hydrodynamic forces due to wave passage:

$$B_z e^{i\gamma_z} = \text{(heave force/wave motion)} \cdot /d/\Delta$$
$$B_x e^{i\gamma_x} = \text{(surge force/wave motion} \cdot d/\Delta$$
$$B_\theta e^{i\gamma_\theta} = \text{(pitch moment/wave motion)} \cdot 1/\Delta$$

displacement, 20 lb; pitch gyradius, 12.2 in.; and roll gyradius, 9.8 in.

Correlation with Free-Model Tests

Description of Free Model

Tests of a dynamic model of a cruciform array of four floats were conducted in regular waves to determine motion response in head seas. The model chosen was adapted from the Convair-proposed P5A float configuration[10] and the tests were conducted under Bureau of Naval Weapons Contract Now 66-0447-d.

Figure 8 is a sketch of the model with a list of some of its important dynamic characteristics. This model was tested in DL Tank 2 (75 ft × 75 ft, 4.5 ft deep), attached to the DL "motions apparatus," a servo-controlled device which permits up to 6 degrees of freedom of unrestrained motion and simultaneous measurement of motion. The results of tests conducted in head seas are compared with computations based on Eqs. (7–9) in Figs. 9–11.

Test records consist of oscillograms of heave, pitch, and surge motions and of wave amplitude measured approximately in line with the longitudinal position of the model's center of gravity.

Description of Calculation Procedure

The motion of the model was determined from Eqs. (7–9) by using the measured forces, acting on a single float, that are due to waves and the float's own motion. Then calculations were carried out with slide rule and polar coordinate graph paper (used for the vector addition).

The symmetric characteristics of this model substantially simplify the equations of motion. For this case especially, the equation for vertical forces contains only the ζ motion as an unknown and the horizontal-force and pitch-moment equations are independent of ζ.

Figure 8 shows that the hull floats and the wing floats have slightly different slenderness ratios as well as different displacements. For the present computations, and for reduction of the experimental work required to characterize the float forces, the wing floats are represented in the computations by floats with the same displacement as the wing floats of the model but with the slenderness ratio of the hull float.

Comparison of Model Tests and Calculations

The predicted heave motion as a function of wave elevation, and its phase relative to the wave motion at the (mean) longitudinal position of the center of gravity, are shown in Fig. 9. Experimental points obtained in the tests of the free model are also shown. The agreement of motion amplitudes is quite good over the entire frequency range. Phase prediction, however, is good only for frequencies up to about $\omega = 5$ rad/sec.

The pitch motion as a fraction of maximum wave slope and its phase are exhibited in Fig. 10 along with the experimental

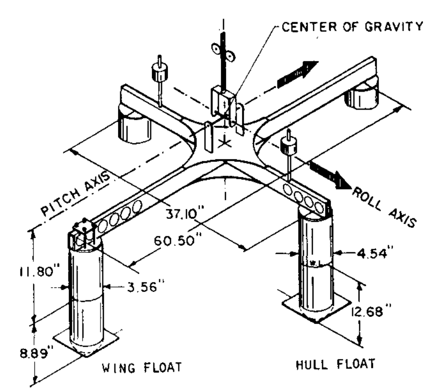

Fig. 8 Cruciform float-platform model.

points. Phase agreement is good but amplitude agreement is not equally good.

Surge-motion results are given in Fig. 11. Fewer experimental data could be analyzed for this case than for pitch and heave since a significant model drift rate required low surge-motion recorder sensitivity. The amplitude agreement is satisfactory but some discrepancies in phase exist.

The pitch and surge motions reported in this paper differ slightly from those presented in the earlier paper[6] where the surge equation and some of the force coefficients were incorrect. The agreement between experiment and calculation is not significantly altered however. The measured and computed phases are somewhat closer, but the pitch motion amplitude agreement is less satisfactory.

Discussion

Experimental data have been obtained for the hydrodynamic forces which act on an isolated float as waves pass by it and as it undergoes various modes of oscillation. This information has been used in the equations of motion for a specific four-float platform with a particular set of inertial qualities to predict the motions of the craft in regular seas over a range of frequencies. Motion predicted by this method compares fairly well with the measured motion of a free model having the same inertial and similar geometric characteristics.

Since the great advantage of the method is that it can provide speedy and accurate motion predictions for a wide variety of vehicle types with differing geometrical features, center-of-gravity locations and moments of inertia, the experimental determination of forces acting on a variety of floats should be undertaken. These force characteristics will be useful as well in predicting loads for structural analysis.

In the present case, the evaluation of the equations of motion was carried out by manual and graphical computation, but this procedure would be better if solution of the equations were programmed for a high-speed digital computer. For arrays of floats which do not possess the symmetrical qualities of the model studied here, this would be quite important. The method should be applied to the study of the motion characteristics of many configurations to facilitate direct comparison with corresponding experimental results.

The procedures should be extended to motion prediction for irregular seas to determine the most useful form for such an analytical development. Since many applications of this kind of flotation system are subject to propulsion forces, gyrodynamic forces associated with rotating machinery, cable load forces, and wind loads, these forces should be incorporated in subsequent analyses and methods of evaluating them should be introduced. Many of these forces are not linearly proportional to the wave elevation or vehicle motion. Methods for treating the nonlinearities should be sought.

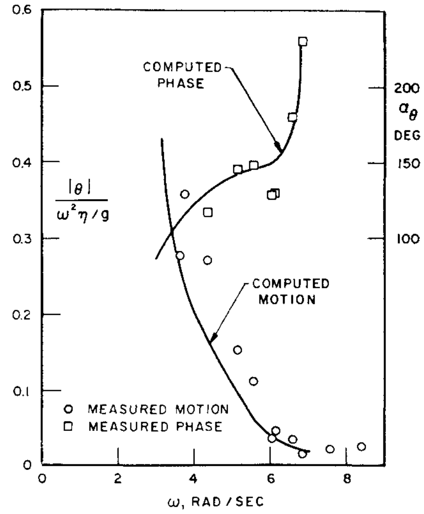

Fig. 10 Measured and computed pitch motion.

The basic procedure (separating the hydrodynamic forces acting on the vehicle into two kinds, viz., those due to wave impingement and those due to float motion) would appear to have wider scope for useful application. The study of certain kinds of one-hulled buoys, for instance, might be facilitated by this approach and methods of evaluating structural loads acting on various kinds of off-shore drilling platforms during deployment and operation might be improved by separate determination of the loads which act on various elements.

The experimental evaluation of the forces which act on the elements is believed to be the important feature of this method since it permits the use of a suitable analytical model of the prototype float element's behavior. In design development, this procedure would probably prove to be substantially better than the usual procedure in which a dynamically scaled model of the entire prototype configuration is tested. If a design is already finalized, however, and the model test program is viewed as simply a "proof test" of its adequacy, the use of the dynamic model may be preferred.

Inasmuch as scale effects are known to occur in the evaluation of hydrodynamic forces, especially with oscillatory damping forces, it is suggested that a series of tests with geometrically similar float models of various sizes be conducted and evaluated.

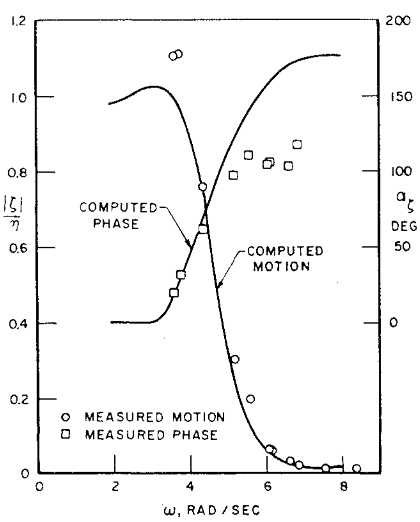

Fig. 9 Measured and computed heave motion.

Conclusions

The proposed method of evaluating the motions of a multi-float platform by means of the equations of motion, in terms of

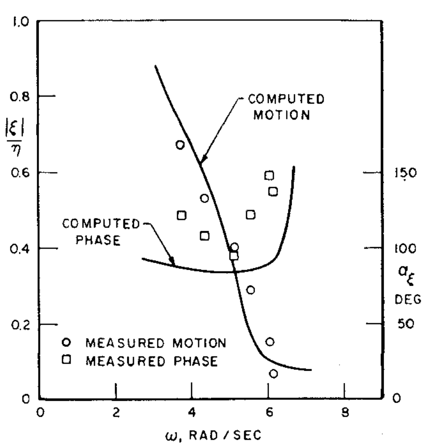

Fig. 11 Measured and computed surge motion.

the wave-exciting forces and float forces due to motions which are evaluated from special tests on a single float, has demonstrated good agreement with tests on a free model. The predicted phase relation of motions to wave passage is not as satisfactory as predicted amplitude and should be further investigated.

This approach should be especially useful in design development and in the structural load evaluation of multifloat vehicles and other types of marine structures. The simple apparatus developed for the tests proved effective and should be further developed for testing at lower frequencies of oscillation.

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Preliminary Design of Hydrofoil Cross Sections as a Function of Cavitation Number, Lift, and Strength

THOMAS G. LANG*
Naval Undersea Research and Development Center, San Diego, Calif.

A set of graphs and equations is developed for quickly determining the minimum-drag form of noncavitating and supercavitating hydrofoils designed for high Reynolds numbers where the boundary layer is fully turbulent. A single classification parameter is derived which simplifies design selection. The results are applicable to the design of propellers, struts, lifting surfaces, and fins for both submerged vehicles and surface craft. It is shown that hydrofoil cross sections can be classified into six basic types of design forms, five of which are cavitating.

Nomenclature†

b = hydrofoil span (L)
c = chordlength of a hydrofoil (L)
C_d = hydrofoil drag coefficient $= D/\rho U^2bc/2$
C_{dc} = cavity drag coefficient
C_{d0} = cavity drag coefficient when $\sigma = 0$
C_f = skin friction drag coefficient
C_L = lift coefficient $= L/\rho U^2bc/2$
C_{L0} = lift coefficient at $\sigma = 0$; $C_{L0} = C_L - 2\sigma$
C_1 = section modulus coefficient $= 2I/t^3c$
D = hydrofoil drag (F)
D_c = cavity drag of a hydrofoil (F)
f = design bending stress, including load factor and factor of safety (FL^{-2})
I = area moment of inertia (L^4)
k = designates the amount of camber of a 2-term hydrofoil camber line
K = hydrofoil classification parameter $= (\sigma - C_L/2)/(M')^{1/2} = \sigma_0/(M')^{1/2} = -C_{L0}/2(M')^{1/2}$
l_c = length of a cavity (L)
L = hydrofoil lift (F)
M = applied bending moment about some cross section of a hydrofoil (FL)
M' = M/fc^3
P = freestream pressure (FL^{-2})
P_v = vapor pressure of the fluid (FL^{-2})
P_1 = minimum pressure on a hydrofoil (FL^{-2})
r = characteristic roughness height (L)
r' = r/c
R = Reynolds number $= Uc/\nu$
t = maximum thickness of a hydrofoil (L)
t' = t/c
t_b = hydrofoil base thickness (L)
t_c = maximum thickness of a cavity (L)
U = freestream velocity (LT^{-1})
U_1 = velocity at the minimum pressure point on a hydrofoil (LT^{-1})
x = chordwise distance to a specific point on a hydrofoil from its leading edge (L)
x' = x/c
y = distance from the chordline to a specific point on a hydrofoil surface (L)
y' = y/c
$y'_l(x')$ = local nondimensional lower-surface height above the chordline

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* Head, Advanced Design Staff, Systems Analysis Group.

† Dimensions are in force F, length L, and time T.